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$$a \left[ x + 2y \left( \frac{dy}{dx} \right)^2 + y^2 \frac{d^2 y}{dx^2} \right] + 2b \left[ y^2 + 4xy \frac{dy}{dx} + x^2 \left( \frac{dy}{dx} \right)^2 + x^2 y \frac{d^2 y}{dx^2} \right] = 0 \dots (2).$$

Eliminating  $a$  and  $b$  in equations (1) and (2), we have

$$\begin{vmatrix} x^2 + y^2 \frac{dy}{dx}, & xy \left( y + x \frac{dy}{dx} \right) \\ x + 2y \left( \frac{dy}{dx} \right)^2 + y^2 \frac{d^2 y}{dx^2}, & \left( y + 2x \frac{dy}{dx} \right)^2 + x^2 y \frac{d^2 y}{dx^2} - \left( \frac{dy}{dx} \right)^2 \end{vmatrix} = 0,$$

which is the Sylvestrian Reciprocant of  $ax^3 + 3bx^2y^2 + ay^3 + d = 0$ , since this function would have the same form if  $x$  were the dependent and  $y$  the independent variable.

167. Proposed by G. B. M. ZERE, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A weight of  $m$  pounds falls and is broken into  $n$  pieces after which it is found that all weights, in pounds, from 1 to  $m$  can be weighed. Find the weight of each piece. Apply when  $m=121$ ,  $n=5$ .

I. Solution by the PROPOSER.

Let  $x_1, x_2, x_3, \dots, x_n$  be the  $n$  pieces. Then  $x_1 + x_2 + x_3 + \dots + x_{n-1} + x_n = m$ ,  
 $x_2 - x_1 = x_1 + 1$ , or  $x_2 = 2x_1 + 1$ .

$$x_3 - x_2 - x_1 = x_2 + x_1 + 1, \text{ or } x_3 = 2x_2 + 2x_1 + 1 = 3(2x_1 + 1) = 3x_2.$$

$$x_4 - x_3 - x_2 - x_1 = x_3 + x_2 + x_1 + 1, \text{ or } x_4 = 2x_3 + 3x_2 = 9(2x_1 + 1) = 9x_2.$$

$$\text{Generally, } x_r = 3^{r-2}(2x_1 + 1) = 3^{r-2}x_2.$$

$$\therefore x_1 + x_2 + x_3 + \dots + x_n = x_1 + (2x_1 + 1)(1 + 3 + 9 + 27 + \dots + 3^{n-2}) = m.$$

$$\therefore x_1 + (2x_1 + 1)(3^{n-1} - 1) = 2m \text{ or } x_1 = \frac{2m + 1 - 3^{n-1}}{2 \cdot 3^{n-1}}.$$

$$x_2 = (2x_1 + 1) = \frac{2m + 1}{3^{n-1}}; \quad x_r = 3^{r-2}x_2 = \frac{2m + 1}{3^{n-r+1}}.$$

When  $m=121$ , and  $n=5$ ,  $x_1=1$ ,  $x_2=3$ ,  $x_3=9$ ,  $x_4=27$ ,  $x_5=81$ .

II. Solution by FRANK L. GRIFFIN, Graduate Student, The University of Chicago.

Let  $f(n)$  = number of groupings of  $n$  weights in two groups; then the maximum number giving one group a preponderance is  $f(n)/2$ .

Now,  $f(n) = 3^n - 1 \dots$  (i) [for proof see below]. Hence, to weigh all weights, in pounds, from 1 to  $m$  by using  $n$  weights, it is necessary that

$$m \leq \frac{3^n - 1}{2} \dots \text{(ii)}.$$

By using the  $n$  weights, 1, 3, 9, ...,  $3^{n-1}$ , all weights, in pounds, from 1 to

their sum,  $S_n \left[ = \frac{3^n - 1}{2} \right]$  can be obtained; *i. e.* from 1 to [or beyond]  $m \dots$  (iii).

[For proof of (iii) also see below].

Therefore, we arrive at the following solution:

A. When  $m = \frac{3^n - 1}{2} = S_n$ .

The  $n$  weights, 1, 3, 9, 27, ...,  $3^{n-1}$ , satisfy both requirements: (a) sum of weights  $= m$ , (b) possibility of obtaining all weights from 1 to  $m$ .

B. When  $m < \frac{3^n - 1}{2}$ .

The  $(n-1)$  weights, 1, 3, 9, ...,  $3^{n-2}$ , together with  $(m - S_{n-1})$  as the other, satisfy requirements (a) and (b).

C. When  $m > \frac{3^n - 1}{2}$  there is no solution.

In the particular case where  $m = 121$ ,  $n = 5$ ,  $m = \frac{3^n - 1}{2}$ , therefore the weights are 1, 3, 9, 27, and 81. [If  $m$  had been between 41 and 121,  $n$  would still have to be as much as 5, but the last weight would have been less than 81, (or  $= m - 40$ )].

PROOF OF (i). Let  $(A) \dots (B)$  be any grouping obtainable with  $n$  weights; then by using one more weight,  $w_{n+1}$ , we get groupings as follows:

$$(A) \dots (B); (A + w_{n+1}) \dots (B); (A) \dots (B + w_{n+1}).$$

Or for  $(n+1)$  weights there are  $3.f(n)$  groupings and in addition the two:

$$(w_{n+1}) \dots (0) \text{ and } (0) \dots (w_{n+1}).$$

$$\therefore f(n+1) = 3.f(n) + 2 \dots (1).$$

Assume  $f(n) = 3^n - 1$ ; then from (1),  $f(n+1) = 3^{n+1} - 1$ . Whence by induction, the formula holds for all values of  $n$  as it is evidently true for  $n=1$ ,  $n=2$ .

PROOF OF (iii). Assume that with  $k$  weights, 1, 3, 9, ...,  $3^{k-1}$ , all weights up to  $S_k \left( = \frac{3^k - 1}{2} \right)$  can be obtained. Then, since  $w_{k+1} = 3^k = 2S_k + 1$ ,

$$S_k + c = w_{k+1} - (s_k + 1 - c).$$

Therefore, any weight  $S_k + c$  between  $S_k$  and  $S_{k+1}$  can be obtained by combining with  $w_{k+1}$  a grouping  $(S_k + 1 - c)$ , which is not less than  $S_k$  and hence is obtainable by weights,  $w_1, w_2, \dots, w_k$ . Therefore by using the first  $(k+1)$  weights, all weights up to  $S_{k+1}$  can be obtained.

Hence, by induction, since by weights  $w_1, w_2$ , all weights up to  $s_2 (= 4)$

can be obtained, all weights from 1 to  $\frac{3^n-1}{2}$  can be obtained by using the weights 1, 3, 9, 27, .... $3^{n-1}$ .

Also solved by J. SCHEFFER, and J. E. SANDERS.  
No solution of problem 168 has yet been received.

### GEOMETRY.

178. Proposed by JOHN M. ARNOLD, Crompton, R. I.

A cylinder thirty feet long and two feet in diameter is to be placed in a machinery car, the inside dimensions of which are eight feet wide and eight feet high. Find length of shortest car that will contain it.

Solution by the PROPOSER.

Consider the car standing on one end, which we shall call the base. Fig. 2.

If a projection of the cylinder be made on the base, each end will be projected into an ellipse with its minor axis on the diagonal  $MN$  of the base.

Fig. 1 is a vertical plane taken on the line  $MN$ . [In Fig. 2, the points  $H$ ,  $D$ ,  $E$ , and  $F$  correspond to the same points in Fig. 1.]

As the sides of the square in Fig. 2 are tangents to the equal ellipses, we have by Analytical Geometry  $MO = \sqrt{A^2 + B^2}$ , where  $A$  and  $B$  are the semi-major and semi-minor axes,

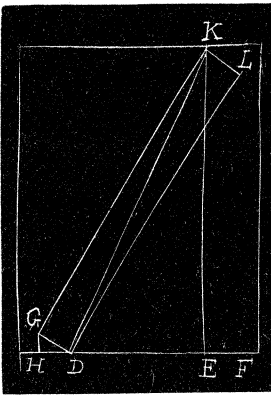


Fig. 1.

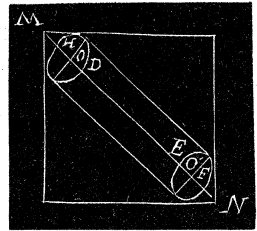


Fig. 2.

$$2MO + OO' = MN = 8\sqrt{2} \dots (1).$$

In the similar triangles  $GDH$  and  $DLF$ ,  $GD : HD = DL : LF$ , or  $2A : 2B =$

$$30 : \frac{30B}{A}. \text{ Then } DF = OO' = \sqrt{900 - \frac{900B^2}{A^2}}.$$

Substituting values of  $OO'$  and  $MO$  in (1),

$$2\sqrt{A^2 + B^2} + \sqrt{900 - \frac{900B^2}{A^2}} = 8\sqrt{2}.$$

$A = \text{radius of cylinder} = 1$ . Substituting and reducing

$$12769B^4 - 21728B^2 = -9184,$$